The Paretian Optimum

However, after a redistribution of the commodities, if the consumers are brought at some point on the contract curve between E and F, then both the consumers would benefit for both of them would reach now higher ICs, and if they are brought just at the point E or F, then one of them will benefit, while the utility level of the other will remain the same.

This shows that any point A, which does not lie on the CCE, is Pareto-non-optimal and by a redistribution of the commodities, if we bring the consumers on to the EF segment of the CCE, then at least one of them would benefit, the utility level of the other remaining the same.

We have seen that all points on the contract curve are Pareto-efficient. However, we cannot compare the points on the contract curve because that will involve interpersonal comparison of utility, which is not possible without an explicit value judgement.

Mathematical Derivation of the Conditions:
We may also derive mathematically the marginal condition for Pareto efficiency in consumption, or,

Exchange. Let us suppose that the utility functions of the two consumers I and II are respectively,

$u_1 = u_1 (q_{21}, q_{12})$
and $u_2 = u_2 (q_{21}, q_{22})$ (21.12)

where q_{11} and q_{12} are the quantities of Q_1 and Q_2 consumed by consumer I and q_{21} and q_{22} are the quantities of the two goods consumed by individual II.

If and q_2 are the given quantities of the two goods, then we have:

$q_{11} + q_{21} = q^0_1$
and $q_{12} + q_{22} = q^0_2$ (21.13)

It is evident from (21.12) that the utility level of each consumer depends only upon the quantities consumed by him and not upon the quantities consumed by the other. That is, it has been assumed here that external effects are absent.

Pareto-efficiency in consumption implies that u_1 is maximised subject to a given $u_2 = u^0_2$, or, the other way round. Let us then form the relevant Lagrange function, V, for the constrained maximisation of u_1 as

$V = u_1 (q_{11}, q_{12}) + \lambda[u_2(q_2(q_{21}, q_{22}) - u^0_2]$

where λ is the Lagrange multiplier.

Now, the first-order conditions for the constrained maximisation of u_1 subject to $u_2 = u^0_2$ are:

$$\left.\begin{aligned}
\frac{\partial V}{\partial q_{11}} &\equiv \frac{\partial u_1}{\partial q_{11}} - \lambda.\frac{\partial u_2}{\partial q_{21}} = 0 \quad [\because q_{21} = q_1^0 - q_{11}] \\
\frac{\partial V}{\partial q_{12}} &\equiv \frac{\partial u_1}{\partial q_{12}} - \lambda\frac{\partial u_2}{\partial q_{22}} = 0 \quad [\because q_{22} = q_2^0 - q_{12}] \\
\frac{\partial V}{\partial \lambda} &\equiv u_2(q_1^0 - q_{11}, q_2^0 - q_{12}) - u_2^0 = 0
\end{aligned}\right] \qquad (21.15)$$

From (21.15), we have:

$$\frac{\dfrac{\partial u_1}{\partial q_{11}}}{\dfrac{\partial u_1}{\partial q_{12}}} = \frac{\dfrac{\partial u_2}{\partial q_{21}}}{\dfrac{\partial u_2}{\partial q_{22}}}$$

$$\Rightarrow MRS_{Q_1,Q_2} \text{ of consumer I} = \Rightarrow MRS_{Q_1,Q_2} \text{ of consumer II} \qquad (21.16)$$

which is the same as (21.11).

Pareto-efficiency condition (21.11) or (21.16) gives us that the given quantities of the two goods should be distributed among the two consumers in such a way that the MRS between the goods may be the same for the two consumers.